



*Chip firing* is a process on a *graph*, which is a network of vertices connected by edges. Initially, a stack of “chips” is placed on each vertex. A vertex is allowed to “fire” its chips if it has at least one chip for each vertex to which it is connected (its “neighbors”). When a vertex fires, it sends one chip to each neighbor. We repeat this process until no vertex has enough chips to fire, which we call a “final configuration.”

The illustration shows chip firing on a cubical grid in 3D. Every vertex has six neighbors, so it needs six chips in order to fire. In this example, every vertex except for the origin started with 4 chips, and the origin started with 100,000 chips. The final configuration forms a cube. Vertices with 0, 1, 2, 3 or 4 chips are rendered as small cubes of different colors. Most of the vertices have 5 chips in the final configuration, so we rendered them as transparent, to exhibit the funnel-like shapes going into the cube from each of its faces. The octant of the cube facing the viewer is also transparent, and only half of the lower octant is shown. This is to show the inner structure of the final configuration, which resembles “leaves.”

A computer-generated rendering allows us to explore both the outer shape and the inner structure of the pattern interactively. Our original plan was to 3D print the patterns inside the object, but they turned out to be too fragmented, so a 3D print would fall apart. Having a digital model allowed for switching on and off the visibility of certain vertices, which made it easy to identify larger structures within the geometry.

Initially, we represented each vertex by its own cubical geometry object in the software Blender, but this yields too many objects, and crashed the Blender GUI. Therefore, we decided to create only six objects, each representing one of the numbers of remaining chips in the final configuration. Using the clipping plane built into the Blender camera, we could also clip the object and thus show any planar slice of the illustration. However, we wanted spherical, not planar clipping. Thus, we decided to build more geometries again: six geometries for each “shell” around the origin. That is, all cubes of a fixed radius distance to the origin and of the same color were collected in one object. By hiding the outside shells, it became possible to interactively show the internal layers of the structure.

*Further information:*  
Caroline J. Klivans, *The Mathematics of Chip-Firing*, CRC Press (2018).



**MARTIN SKRODZKI,  
CAROLINE J. KLIVANS,  
AND PEDRO F.  
FELZENSZWALB**

ICERM; Brown University;  
Brown University

*computer-generated graphic*