Three-dimensional skin-coloring of a four-dimensional being

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Why do tigers and zebras have stripes while other animals like cows or leopards are spotted? This question is one of the aspects of the broader task to understand *morphogenesis*. Composed of the two Greek words *morphé* (shape) and *génesis* (creation), the term describes the biological process of an organism developing its shape. One of the earliest authors to investigate this question was d'Arcy Wentworth Thompson, who devoted his 1917 treatise "On Growth and Form" to it, see Thompson 1917. Another notable contribution was made by Alan M. Turing in his article "The chemical basis of morphogenesis", issued in the philosophical transactions of the royal society of London, see Turing 1952. While Thompson's approach to morphogenesis is largely based on different growth rates of the animal, Turing focuses—as suggested by the title—on a chemical mechanism giving rise to different skin color patterns. He states in the abstract of his article "that a system of chemical substances, called morphogens, reacting together and diffusing through a tissue, is adequate to account for the main phenomena of morphogenesis," Turing 1952, p. 37.

Indeed, though much later, the patterns predicted by Turing have been found in biological settings and physical systems. While they indeed describe certain animal skin patterns, they surprisingly also arise in larger biological phenomena, like the formation of termite hills.

With the growing availability of computers, discretizations of models become increasingly important. Regarding the concept of Turing patterns, a most notably contribution was made by David A. Young in his 1984 paper "A local activator-inhibitor model of vertebrate skin patterns," see Young 1984. His model is not only discrete, but also reduces Turing's setup to two simple morphogens with clear functionalities: one activator morphogen that causes cells to be differentiated (colored) and one inhibitor morphogen that prevents the differentiation of cells. The structures obtained from Young's discretizations of Turing's work are consequentially referred to as Turing-like patterns. While Young does never state it in his paper, he basically gave the description of a two-dimensional cellular automaton to produce the patterns.

The formulation of Turing-like patterns by Young allows for a very efficient evaluation of different parameters and setups. While Turing has "obtained [his results] in a few hours by a manual computation" Turing 1952, p. 59, modern computers can create respective imagery within seconds. This makes these patterns interesting for different applications. For instance, an article has been devoted to generalizations of the two-dimensional patterns to three-dimensional structures, see Skrodzki and Polthier 2017. The resulting three-dimensional patterns in particular pose a visualization challenge: It is no longer feasible to render each cell as a colored solid as the outermost cells would obscure the view to the inside of the pattern. To illustrate the obtained findings, only the borders between differentiated and undifferentiated cells were shown.

The submitted video explores such a three-dimensional sculpture that has also been 3D-printed. Following from the model, this sculpture represents the three-dimensional skin coloring of a four-dimensional being. The sculpture consists of several "pipe-like" shapes which are one of seven possible statuses the three-dimensional model can attain, see Skrodzki, Reitebuch, and Zimmermann 2020.

References

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