H. Lipschütz ${ }^{1}$, U. Reitebuch ${ }^{1}$, K. Polthier ${ }^{1}$, M. Skrodzki ${ }^{2}$<br>${ }^{1}$ FU Berlin and ${ }^{2}$ TU Delft

## Isotropic Point Cloud Meshing using unit Spheres (IPCMS)

## Contributions

Our contributions are (full details to be found in [1]):

- presentation of a meshing algorithm that places touching spheres of uniform radius on the input,
- which creates edge lengths close to uniformity and of a guaranteed minimum length,
- as well as manifold output, provided a suitable input geometry and good enough normals.


## Theory

Let $\mathcal{M}$ be an orientable, compact, closed $\mathcal{C}^{2}$-manifold embedded into $\mathbb{R}^{3}$ of finite reach $\rho:=\inf \left\{\|a-m\| \mid a \in \mathcal{A}_{\mathcal{M}} \wedge m \in \mathcal{M}\right\} \in \mathbb{R}_{>0}$, where $\mathcal{A}_{\mathcal{M}}$ is the medial axis of $\mathcal{M}$ consisting of the points $q \in \mathbb{R}^{3}$ fulfilling $\min _{p \in \mathcal{M}}|q-p|=|q-\hat{p}|=|q-\tilde{p}|$ for $\hat{p} \neq \tilde{p} \in \mathcal{M}$
Lemma: Let $\boldsymbol{p} \in \mathcal{M}$ be a point and let $\boldsymbol{N}_{\boldsymbol{p}}$ denote its normal. Then, for $\boldsymbol{r}<\boldsymbol{\rho}$, the image of $\boldsymbol{B}_{r}(\boldsymbol{p}) \cap \mathcal{M}$ under the projection $\boldsymbol{\pi}$ in direction of $\boldsymbol{N}_{\boldsymbol{p}}$ to the tangent plane $\boldsymbol{T}_{p} \mathcal{M}$ is a convex set.


## Disk Growing



## Algorithm

Input: point cloud $\mathcal{P}$, normal field $\mathcal{N}$, target edge length $\boldsymbol{d}$, splat size $s$, starting vertices $q, q^{\prime}$, window size $w$, maximum border length $\partial_{\text {max }}$
Output: triangle mesh $\mathcal{T}$ with edge lengths close to uniformity
Build box grid, register splats in all boxes up to distance $\boldsymbol{d}$ (possibly with individual splat sizes), filter points by the average normal, compute box normals. Project $\boldsymbol{q}$ and $\boldsymbol{q}^{\prime}$ to their closest splats, start a graph $\mathcal{G}$ by adding the projections as vertices.
Compute initial vertex candidates and their priority, add candidates to the queue.
 while candidate vertex $\boldsymbol{v}_{\boldsymbol{c}}$ exists in the queue do
if $\mathcal{G}$ has vertex $\boldsymbol{v}$ s.t. $\left\|\boldsymbol{v}-\boldsymbol{v}_{\boldsymbol{c}}\right\|_{2}<\boldsymbol{d}$ or $\boldsymbol{v}_{\boldsymbol{c}}$ fails the projection check then Discard $\boldsymbol{v}_{\boldsymbol{c}}$ else if priority of $\boldsymbol{v}$ is not correct then

Correct priority by pushing $\boldsymbol{v}_{\boldsymbol{c}}$ back to the queue. else

Add $\boldsymbol{v}_{\boldsymbol{c}}$ and edges to its parent vertices to $\mathcal{G}$, update region borders Compute new vertex candidates around $\boldsymbol{v}_{\boldsymbol{c}}$ and their priorities. Add them to the queue.
se

for each region $\boldsymbol{R} \in \mathcal{R}$ do
while length of region border $\partial \boldsymbol{R}$ is $\geq \mathbf{3}$ and $\partial_{\text {max }}$ do
Cut triangle $\boldsymbol{t}$ at the smallest border angle. Add $\boldsymbol{t}$ to $\mathcal{T}$.

## Results



Bowl chinese model [2].


Triangulated graph.


Uniform splat size.


Adaptive splat size.


## References

[1] H. Lipschütz, U. Reitebuch, K. Polthier, and M. Skrodzki. Isotropic Point Cloud Meshing using Unit Spheres (IPCMS). arXiv preprint arXiv:2305.07570, 2023.
[2] Z. Huang, Y. Wen, Z. Wang, J. Ren, and K. Jia. Surface reconstruction from point clouds: A survey and a benchmark. arXiv preprint arXiv:220502413, 2022.

