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SFB TRR 109 Discretization in Geometry and Dynamics

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Large-scale Evaluation of Neighborhood Weights and Sizes

The Experiment

We investigate the quality of weighted neighborhoods $\mathcal{N}_i \subset [n]$ with different sizes, see [1] for complete results. The weights are determined by the normal similarity between points and are given by a sigmoid function modeling both continuous and sharp increases. Our large-scale analysis consists of more than $1,000$ point sets $P = \{p_i \mid i \in [n]\}$, with neighborhoods $\mathcal{N}_i \subset [i]$ and normals $n_i \in \mathbb{S}^2$.

Sigmoid Weights

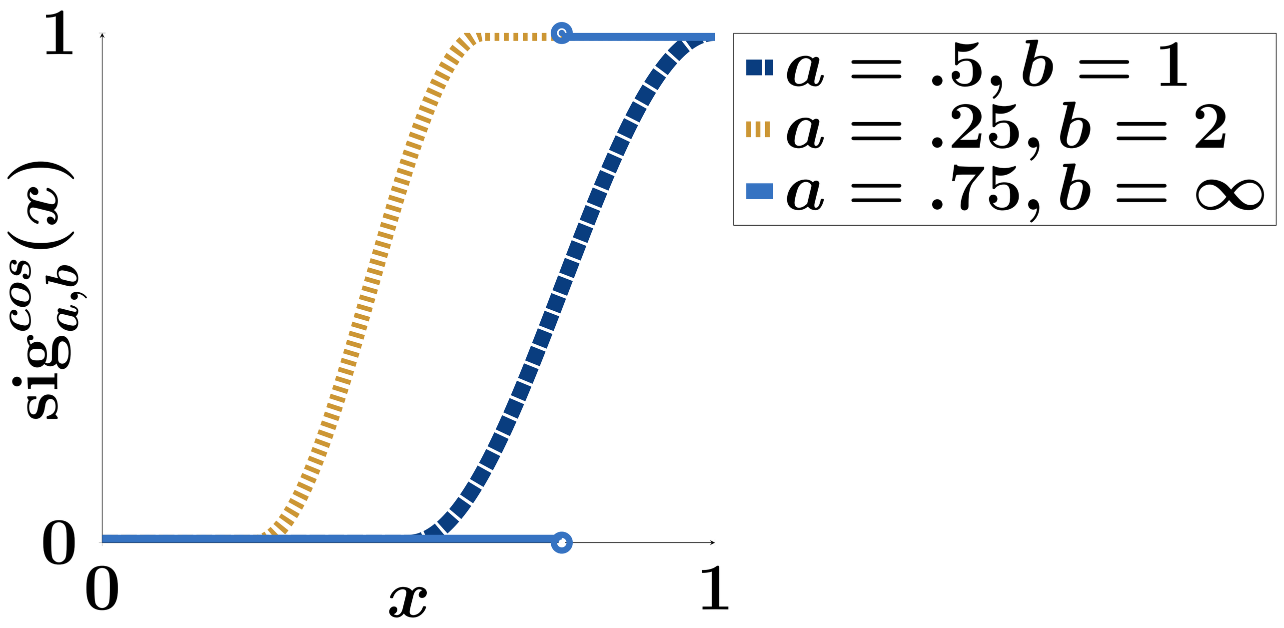


Figure: Plots of the sigmoid $\mathrm{sig}_{a,b}^{cos}(x)$ for three parameter choices.

For $x = \frac{\langle n_i, n_j \rangle + 1}{2}$ and $j \in \mathcal{N}_i$ assign weights for p_j from p_i as

$$w_{ij} = \mathrm{sig}_{a,b}^{cos}(x) = \begin{cases} 0 & x \in (-\infty, a), \\ -\frac{1}{2}\cos\left(\frac{b\pi(x-a)}{1-a}\right) + \frac{1}{2} & x \in [a, a'), \\ 1 & x \in [a', +\infty), \end{cases}$$

with $a \in [0,1)$, $b \in \mathbb{R}_{\geq 1} \cup \{\infty\}$ and $a' = (1-a)b^{-1} + a$.

Evaluation Model

Following [2,3], consider the covariance matrix of $\mathcal{N}_i$

$$C_i := \sum_{j \in \mathcal{N}_i} w_{ij}(p_j - \bar{p}_i)(p_j - \bar{p}_i)^T.$$

The eigenvalues $\lambda_i^1 > \lambda_i^2 \geq \lambda_i^3 \geq 0$ of C_i give quantities

$$L_i^\lambda = (\lambda_i^1 - \lambda_i^2)/\lambda_i^1 \quad \text{(linearity)},$$
$$P_i^\lambda = (\lambda_i^2 - \lambda_i^3)/\lambda_i^1 \quad \text{(planarity)},$$
$$S_i^\lambda = \lambda_i^3/\lambda_i^1 \quad \text{(scattering)}.$$

For each point p_i, evaluate these via a Shannon-type error

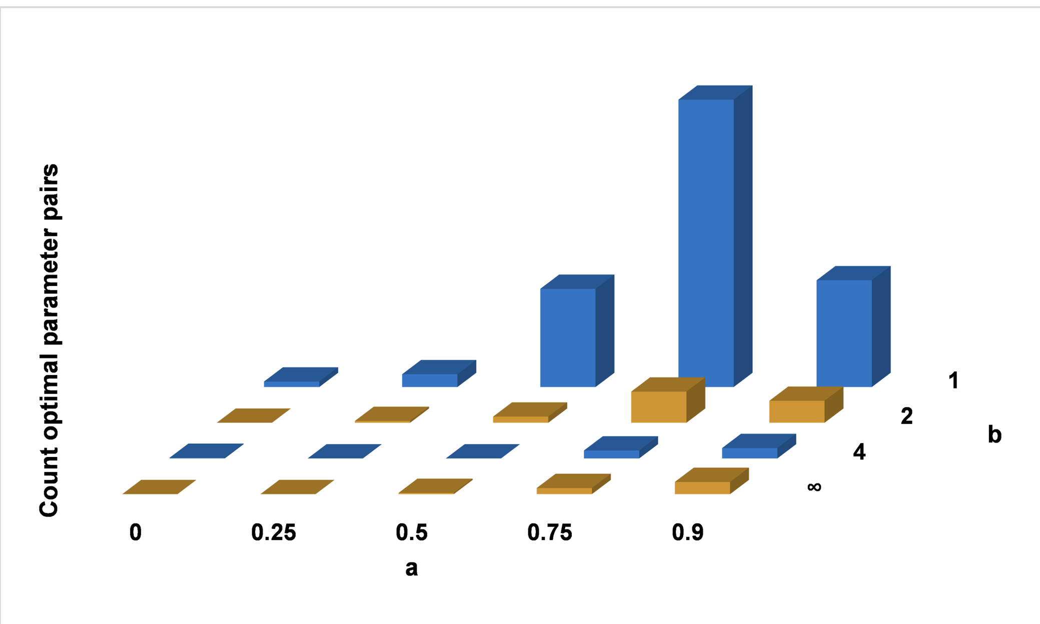
$$E_i^{\mathrm{dim}} = -L_i^\lambda \ln(L_i^\lambda) - P_i^\lambda \ln(P_i^\lambda) - S_i^\lambda \ln(S_i^\lambda)$$

and for a range of possible neighborhood sizes $k \in \mathfrak{K} := \{6, \dots, 20\}$, determine the optimal weighting parameter pair (a^*, b^*) via

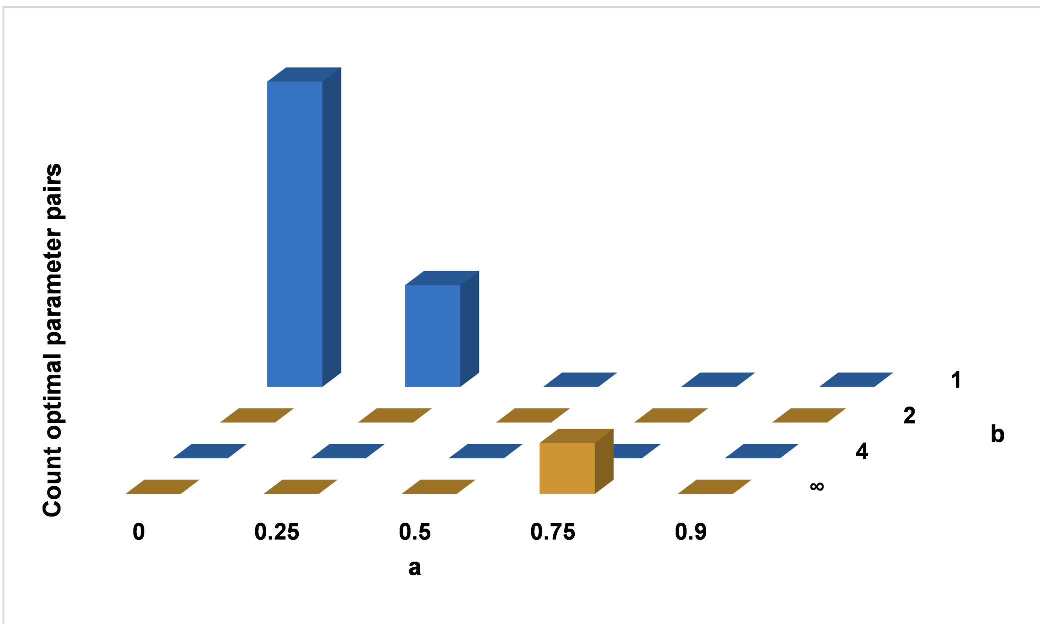
$$(a^*, b^*) = \arg\min_{(a,b) \in \mathfrak{P}} \frac{1}{|P|} \sum_{i=1}^{|P|} \min_{k \in \mathfrak{K}} E_i^{\mathrm{dim}}. \quad (1)$$

Optimal Parameter (a^*, b^*) analysis

We run the evaluation from Equation (1) on $1,000$ clean [4] and nine noisy models [5].



(a) $1,000$ clean models [4].



(b) nine noisy models [5].

Figure: Distribution of optimal parameters (a^*, b^*) on the two model sets.

Could the values be increased $(a+, b+)$, were they maximal $(a = .9, b = \infty)$, or would an increase lead to a faulty evaluation, e.g. via empty neighborhoods $(\neg a+, \neg b+)$.

	$a = .9$	$a+$	$\neg a+$	$b = \infty$	$b+$	$\neg b+$
clean models [4]	248	20	732	32	968	0
noisy models [5]	0	1	8	1	8	0

Table: Distribution into maximal parameters, possible increase, and failure when increasing.

Conclusions:

- Set b small, which takes all neighbors into account.
- Set a as large as possible to only weight similar neighbors highly.
- Have to choose a small enough to actually obtain a non-empty neighborhood.
- Equal-weights $(a = 0, b = \infty)$ or sharp cut-off weights $(b = \infty)$ as widely used in the literature are rarely optimal.

Optimal parameter k analysis

Plot k that achieve optimality in Equation (1).

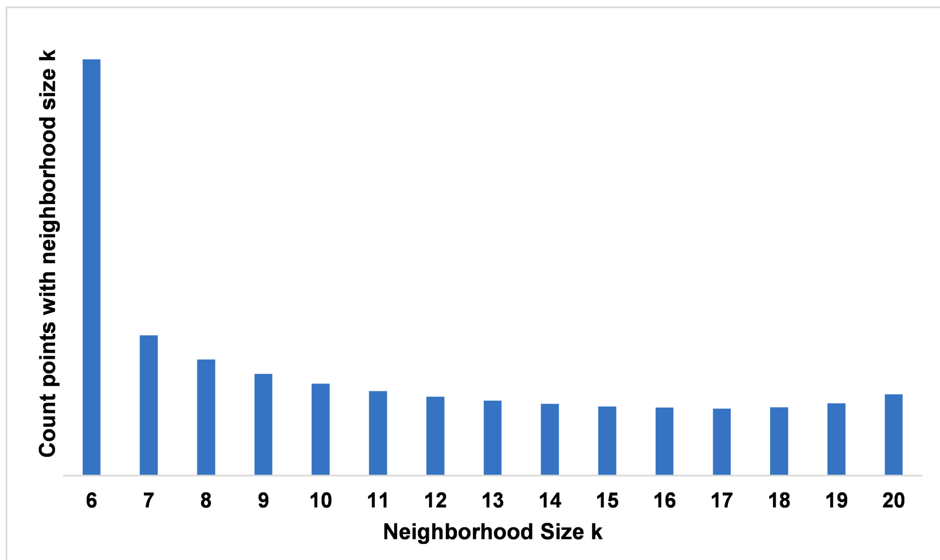


Figure: k distributed over $1,000$ clean models [4].

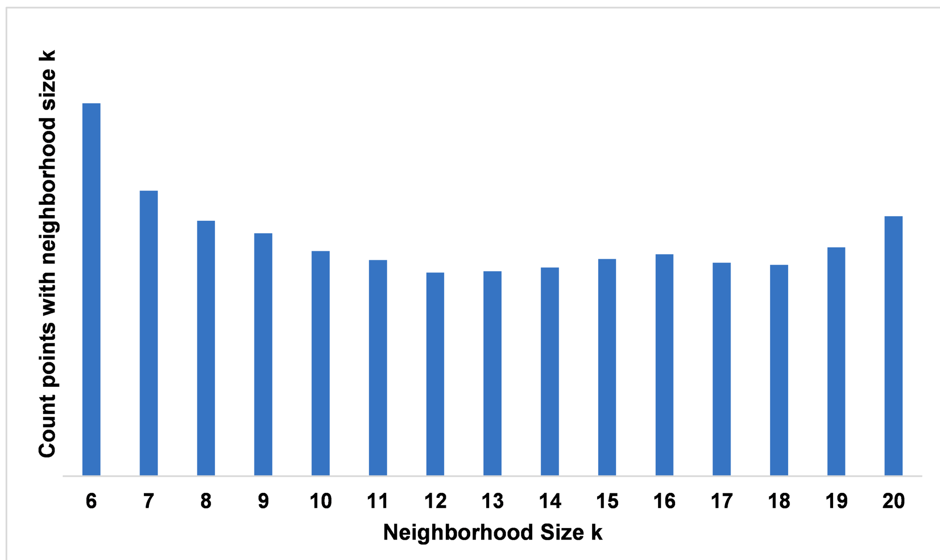


Figure: k distribution over nine noisy models [5].

Conclusions:

- Use small k for clean models, have to vary k for noisy models.
- Optimality is only achieved with varying neighborhood size k, but literature usually fixes some k.

References

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